CSE 210: Computer Architecture
Lecture 21: Floating Point

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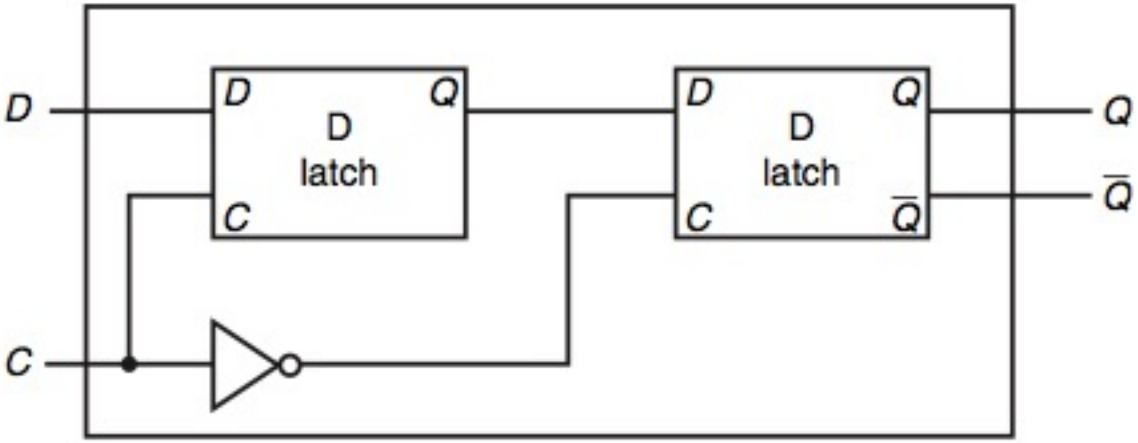
Slides from Cynthia Taylor

Announcements

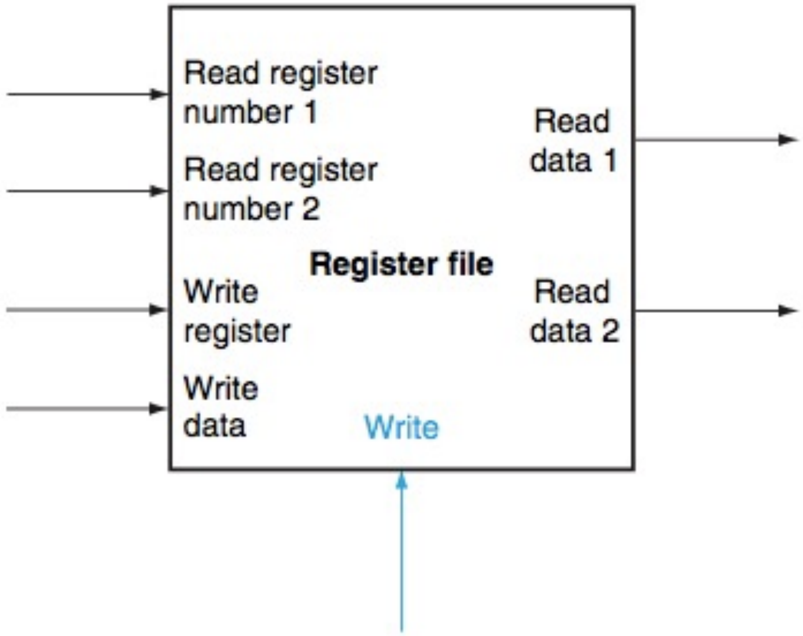
- Problem Set 6 due today
- Lab 5 due a week from Sunday
- Office Hours today 13:30 – 14:30

Registers

- Each 32-bit register will consist of 32 1-bit D flip-flops

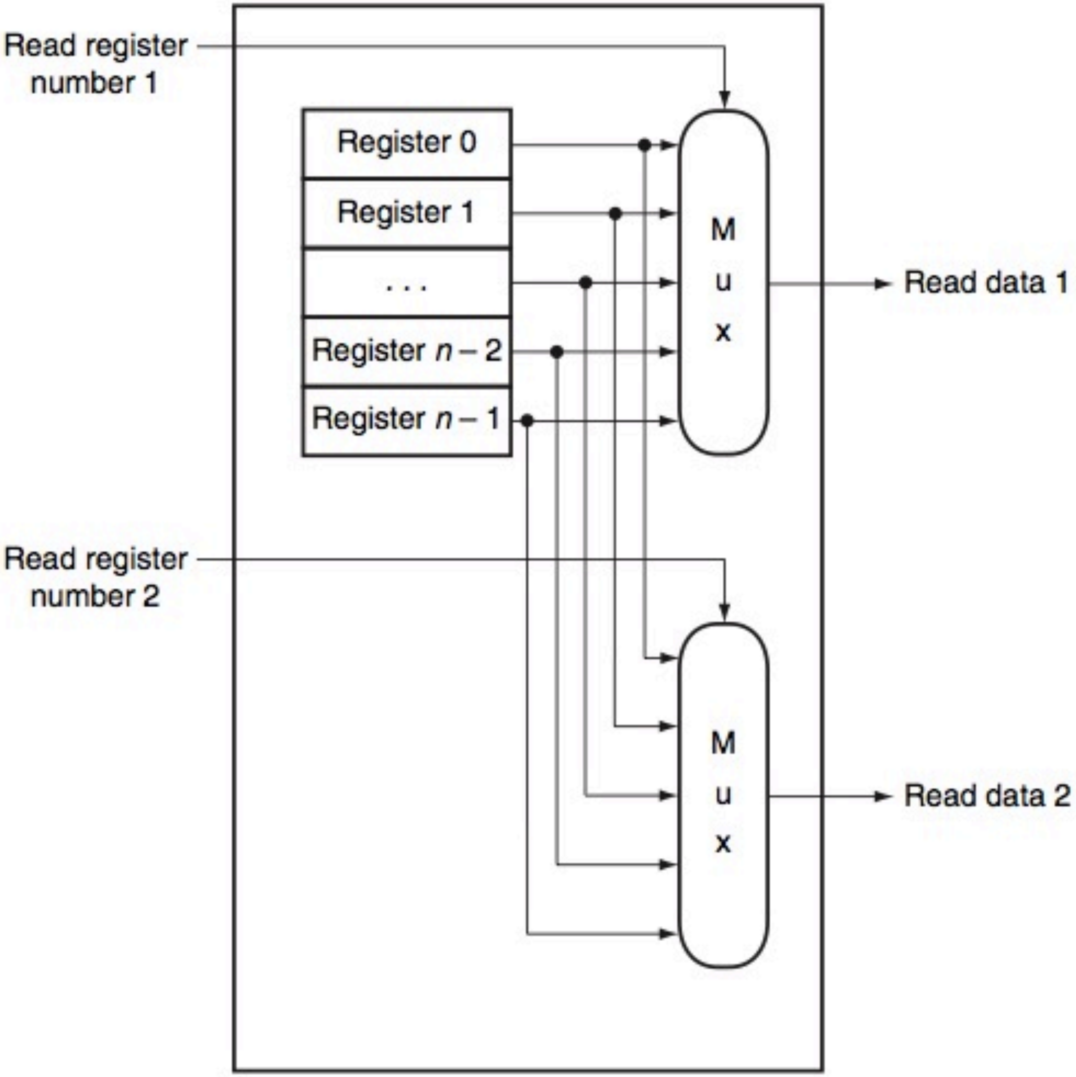


Register File

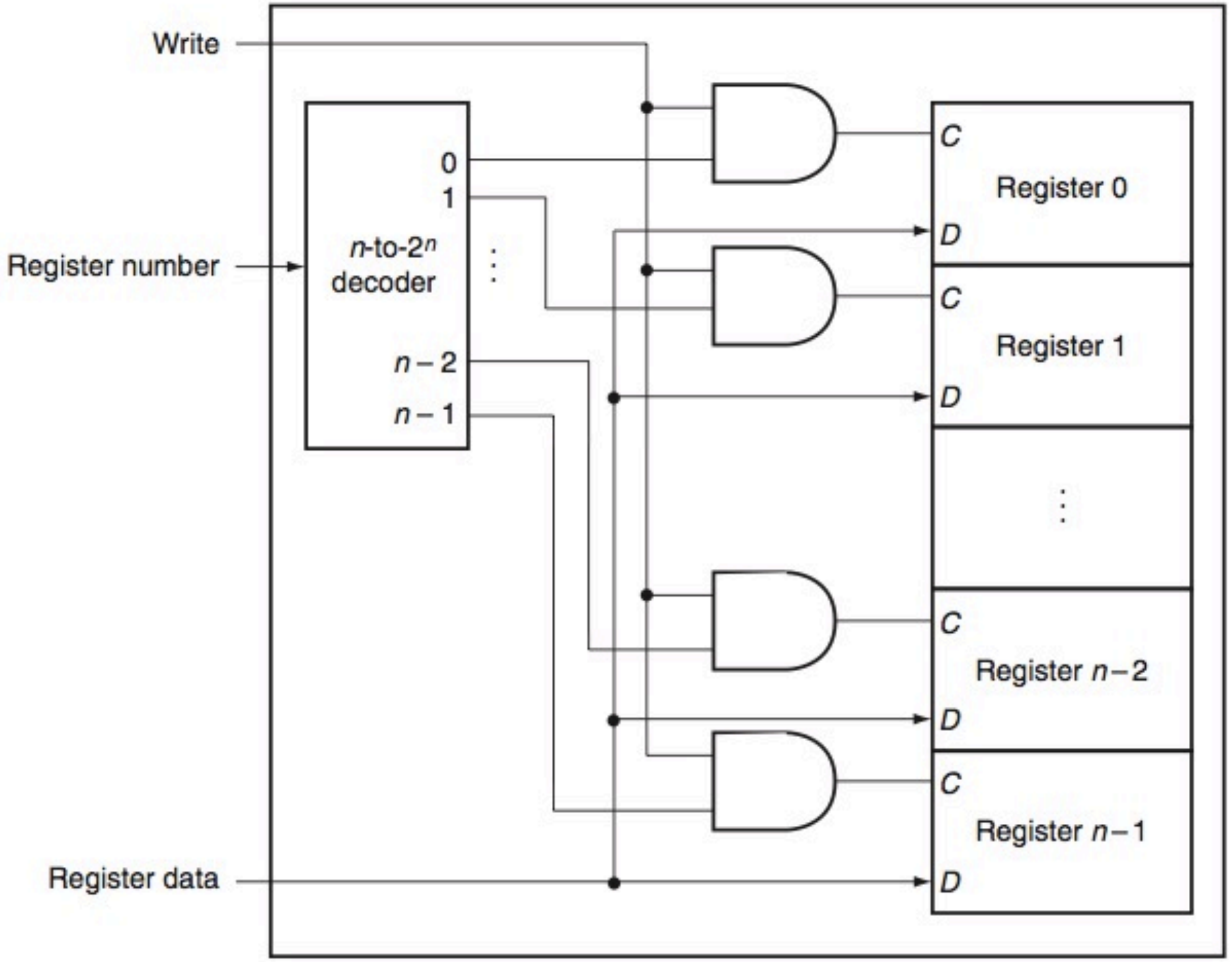


- Set of registers that can be written/read by supplying a register number

Read Function



Write Function



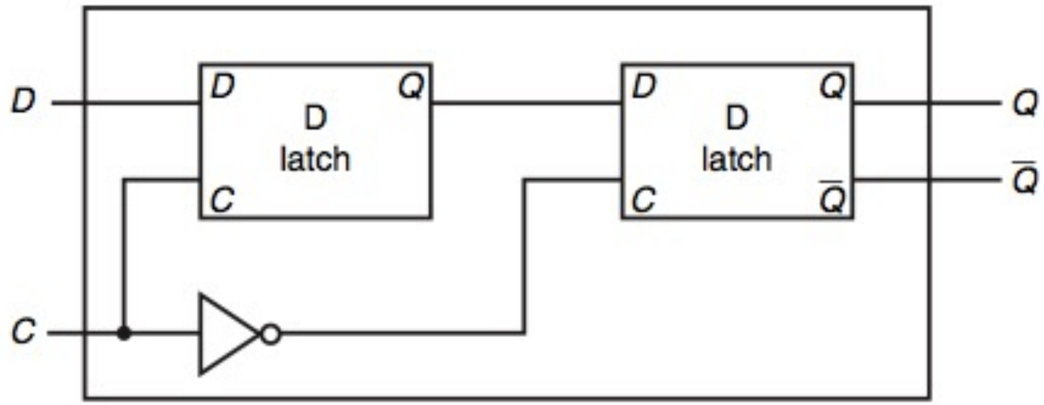
What will happen if we read and write to a register in the same clock cycle?

A. The read will get the previous value

B. The read will get the just written value

C. It is ambiguous

D. None of the above



Floating Point

- Problem: Need a way to store non-integer values

- Including very large, very small and very negative values

How Humans Do This

- Scientific Notation
  - $1.2825 * 10^{2}$
  - $2.004 * 10^{38}$
  - $3.74 * 10^{-27}$
  - $-7.888889 * 10^{40}$

- Normalized Form
  - Always multiply by power of 10
  - Always 1 digit before the decimal point

How Computers Do This

- Floating Point Notation
  - $1.11_2 \times 2^2$
  - $1.0101_2 \times 2^{127}$
  - $1.110001_2 \times 2^{-126}$
  - $-1.0001_2 \times 2^{80}$

- Normalized Form
  - One digit before decimal
  - Multiplied by power of two

101.10001_2

- $101.10001_2 = 2^2 + 2^0 + 2^{-1} + 2^{-5}$

- Integer part is $101_2 = 4 + 1 = 5$

- Fractional part is $0.10001_2 = 1/2 + 1/2^5 = 0.503125$

- Total is 5.503125

We know $101.10001_2 = 5.503125$. What is $1.0110001_2 \times 2^2$

A. 1.37578125

B. 5.503125

C. 22.0125

D. None of the above

–17.125 in binary

- Step 1. Convert integer part: 17 = 10001_2
- Step 2. Convert fractional part: .125 = 1/8 = 0.001_2
- Step 3. Add integer and fractional parts: 17.125 = 10001.001_2
- Step 4. Normalize: $10001.001_2 = 1.0001001_2 \times 2^4$
- Step 5. Add sign: $–17.125 = –1.0001001_2 \times 2^4$

–0.75 in Binary is

A. $-1.1_2 \times 2^{-1}$

B. $-1.1_2 \times 2^{-2}$

C. $-1.001011_2 \times 2^{-1}$

D. $-1.001011_2 \times 2^{-2}$

E. None of the above

$1.2825 * 10^2$ in Binary is

A. $1.000000001_2 \times 2^{-7}$

B. $1.000000001_2 \times 2^6$

C. $1.1001000011001_2 \times 2^6$

D. $1.000000001_2 \times 2^7$

E. None of the above

Want to Represent $(-1)^s * 1.x * 2^e$ in 32 bits

- Divide up 32 bits into different sections

- 1 bit for sign s (1 = negative, 0 = positive)

- 8 bits for exponent e

- 23 bits for significand 1.x

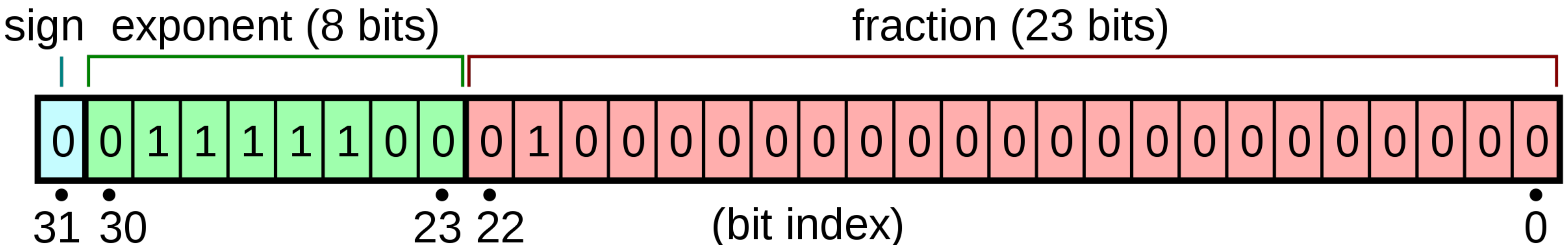
Goal: Get the most out of 32 bits

- The first number before our ~~decimal~~ binary point is always 1
  - – $1.0001 * 2^{4}$
  - – $-1.1011 * 2^{-16}$

- We don’t need to represent it in our remaining 23 bits—it is implicit!

$(-1)^s * 1.x * 2^e$

- 1 bit for sign s (1 = negative, 0 = positive)
- 8 bits for exponent e
- 0 bits for implicit leading 1 (called the “hidden bit”)
- 23 bits for significand (without hidden bit)/fraction/~~mantissa~~ x



sign	exponent (8 bits)	fraction (23 bits)
0	0 1 1 1 1 1 0 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

1.001100101 * 2^7 as a single word

- 1.001100101 * 2^7 as a single word becomes
  - – Sign = 0 (positive)
  - – Exponent = 00000111
  - – Significand = 00110010100000000000000

If we gave more bits to the exponent, and fewer to the fraction, we could represent

A. Fewer individual numbers

B. More individual numbers

C. Numbers with greater magnitude, but less precision

D. Numbers with smaller magnitude, but greater precision

Want To Make Sorting Easy

- Can easily tell if number is positive or negative
  - Just check MSB bit

- Exponent is in higher magnitude bits than the fraction
  - Numbers with higher values will look bigger
  - 0 00000111 10000000000000000000000 = 1.1 * 2^7
  - 0 00001000 10000000000000000000000 = 1.1 * 2^8

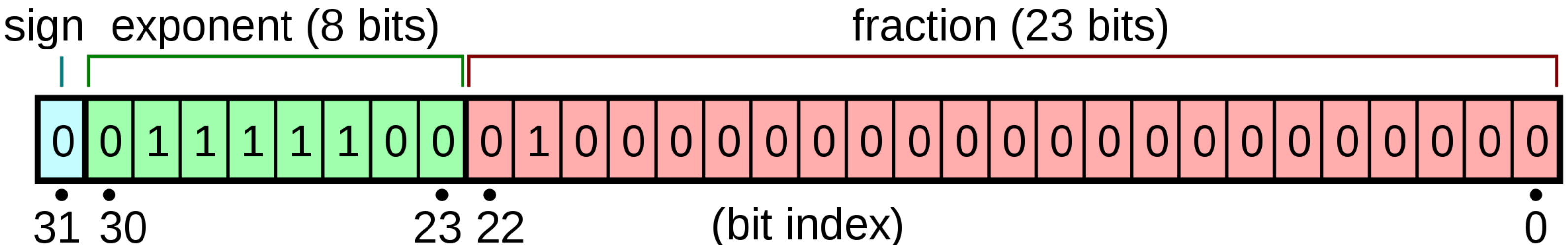
Problem with Two's Compliment

- 0 00000111 10000000000000000000000 = 1.1 * 2^7
- 0 00001000 10000000000000000000000 = 1.1 * 2^8
- 0 11111000 10000000000000000000000 = 1.1 * 2^{-8}

- Solution: Get rid of negative exponents!
  - We can represent 2^8 = 256 numbers: normal exponents -126 to 127 and two special values for zero, infinity, (and NaN and subnormals)
  - Add 127 to value of exponent to encode it, subtract 127 to decode

$(-1)^s * 1.x * 2^e$

- 1 bit for sign s (1 = negative, 0 = positive)
- 8 bits for exponent e + 127
- 0 bits for implicit leading 1 (called the “hidden bit”)
- 23 bits for significand (without hidden bit)/fraction/~~mantissa~~ x



sign exponent (8 bits) fraction (23 bits)

0 0 1 1 1 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 = 0.15625

31 30 23 22 (bit index) 0

1.000000001 * 2^7 in Floating Point

A. 0 00000111 00000000100000000000000

B. 0 00000111 10000000010000000000000

C. 0 10000110 00000000100000000000000

D. 0 10000110 10000000010000000000000

E. None of the above

Reading

- Next lecture: Floating Point

- Problem Set 6 due today

- Lab 5 due a week from Sunday